## ASSIGNMENT SET - I

## Department of Mathematics

# Mugberia Gangadhar Mahavidyalaya 



## B.Sc Hon.(CBCS)

Mathematics: Semester-VI
Paper Code: DSE4T
[Mathematical Modelling]
Answer all the questions

1. What is Monte Carlo Simulation?
2. Use the middle - square method to generate 2 random numbers considering the seed $\boldsymbol{x}_{0}$ $=3043$.
3. Find $\boldsymbol{L}^{-1}\left\{\frac{s+2}{s^{2}(s+3)}\right\}$.
4. Show that Laplace transform of the function $\mathrm{f}(\mathrm{t})=\boldsymbol{t}^{n},-1<\mathrm{n}<0$, exists, but it is not piecewise continuous on every finite subinterval in the range $t \geq 0$.
5. The cost of any non basic variable can be reduced without limit affecting the optimal basic feasible solution to the LPP. Justify
6. If $\mathrm{L}\{\mathrm{f}(\mathrm{t})\}=\frac{50 s+3}{s^{4}+3 s^{2}+(k-4) s}$ and $\lim _{t \rightarrow \infty}(\boldsymbol{t})=1$ then find the value of k where k is a constant .
7. Let $f(t)$ be the continuous function on $[0, \propto]$ whose Laplace transform exists. if $f(t)$ satisfies $\int_{0}^{t}(\mathbf{1}-\boldsymbol{\operatorname { c o s }}(\boldsymbol{t}-\boldsymbol{u})) \mathrm{f}(\mathrm{u}) \mathrm{du}=\boldsymbol{t}^{4}$ then find $\mathrm{f}(\mathrm{t})$.
8. Is the solution $\left(\mathbf{1}, \frac{\mathbf{1}}{\mathbf{2}}, \mathbf{0}, \mathbf{0}, \mathbf{0}\right)$ a basic solution of the equations
$\begin{array}{llll}x_{1} & +2 x_{2} & +x_{3} & +x_{4} \\ x_{1}+2 x_{2}+\frac{1}{2} x_{3}+x_{5}=2 & & \end{array}$
9. What is meant by singularity of a linear ordinary differential equation?
10. Write the general expression of $\boldsymbol{p}_{\boldsymbol{n}}(\mathrm{z})$.
11. What do you mean cycling in linear congruence?
12. Prove that $\boldsymbol{L}^{-1}\left\{\frac{f(s)}{s^{2}}\right\}=\int_{0}^{t} \int_{0}^{v} \boldsymbol{F}(\boldsymbol{u})$ dudv.
13. Does the Laplace transform of $\frac{\text { cosat }}{\boldsymbol{t}}$ exist?
14. What is probabilistic process?
15. Why we generate random number?
16. How do you generate random numbers between 0 and 1 that follow uniform distribution using linear congruence method?
17. Using Monte Carlo simulation, write an algorithm to calculate that party of the volume of an ellipsoid
$\frac{x^{2}}{2}+\frac{y^{2}}{4}+\frac{z^{2}}{8} \leq 16$
18. In the LPP

Maximize $\mathrm{Z}=3 \boldsymbol{X}_{\mathbf{1}}+5 \boldsymbol{X}_{\mathbf{2}}$
Subject to $\boldsymbol{x}_{\mathbf{1}}+\boldsymbol{x}_{\mathbf{2}} \leq \mathbf{1}$
$2 x_{1}+3 x_{2} \leq 1$
$x_{1}, x_{2} \geq 0$
Obtain the variation of $\boldsymbol{c}_{\boldsymbol{j}}(\mathrm{j}=1,2)$ with out changing the optimality of solution.
19. Find $L^{-1}\left\{\frac{1}{((p-a) \sqrt{p}}\right\}$ by the convolution integral .
20. Generate 15 random numbers using middle - square method taking $\boldsymbol{x}_{0}=3043$.
21. Prove the final value theorem $\boldsymbol{L} \boldsymbol{t}_{\boldsymbol{t} \rightarrow \infty} \mathrm{f}(\mathrm{t})=\boldsymbol{L} \boldsymbol{t}_{\boldsymbol{s} \rightarrow \infty} \mathrm{Sf}(\mathrm{S})$.
22. (i) Use Laplace transform to solve the following initial value

$$
\begin{gathered}
\text { problem } \\
\frac{d^{2} y}{d x^{2}}-3 \frac{d y}{d x}+2 y=h(t), y(0)=0, y(0)=0 \text { where } \\
h(t)=\left\{\begin{array}{lr}
2, & 0<t<4 \\
0, & t>4
\end{array}\right.
\end{gathered}
$$

(ii) Prove that $\mathrm{L}\{\sinh a t \cosh a t\}=\frac{a\left(s^{2}-2 a^{2}\right)}{s^{4}+4 a^{4}}$
23. (i) Write a Monte Carlo simulation algorithm for a harbor with unloading facilities for ships finding the answers of the following questions:

1. What is the average and maximum times per ship in

The harbour?
2. What are the average and maximum waiting times per ship?
3. What percentage of the time are the unloading facilities Idel?
(ii) What are the disadvantages of linear congruence method to Generate random numbers?
24. (i) Find the Laplace inverse of the function $\frac{1}{(p+a)^{3}}$
(ii) Solve the following LPP using simplex method

Maximize $Z=6 x+4 y$
Subject to $-x+y \leq 12$
$x+y \leq 24$
$2 x+5 y \leq 80$
$X, y \geq 0$
25. Find the solution of the Bessel differential equation of

Order $\lambda$ at the neighbourhood of $x=0$. Discuss the case

When $\lambda=0$.
26. Describe Monte Carlo Algorithm to find the area under a curve. Using Monte Carlo simulation, write an algorithm to calculate that part of volume of an ellipsoid $\frac{x^{2}}{2}+\frac{y^{2}}{4}+\frac{z^{2}}{8} \leq 16$ that lies on the first octant $x \geq 0, y \geq 0, z \geq 0$
27. Write the proper equation of linear congruence method by which random number is generated. Using this method, find five random number between 50 and 100.
28. What are the disadvantages of middle square method to generate random numbers? Generate 5 random numbers using this method and initially taking $x_{0}=653217$.
29. Two different products, P1 and P2, can be manufactured by one or both of two different machines, M1 and M2. The unit processing time of either product on either machine is the same. The daily capacity of machine M1 is 200 units (of either P1 or P2, or a mix of both), and the daily capacity of machine M2 is $\mathbf{2 5 0}$ units. The shop supervisor wants to balance the production schedule of the two machines such that the total number of units produced on one machine is within 5 units of the number produced on the other. The profit per unit of P 1 is $\mathbf{\$ 1 0}$ and that of P 2 is $\mathbf{\$ 1 5}$. Set up the problem as an LP in equation form.
30. Solve the following LPPs using simplex method:

Maximize $z=5 x_{1}-2 x_{2}+3 x_{3}$

Subject to

$$
\begin{aligned}
& 2 x_{1}+2 x_{2}-x_{3} \geq 2 \\
& 3 x_{1}-4 x_{2} \leq 3 \\
& x_{2}+3 x_{3} \leq 5 \\
& x_{1}, x_{2}, x_{3} \geq 0 .
\end{aligned}
$$

31. Solve the following LPP by graphical method Maximize

$$
60 x+50 y
$$

$$
x+2 y \leq 1000
$$

Subject to $\quad 4 x+2 y \leq 1600$

$$
x, y \geq 0
$$

32. Discuss the sensitivity of changes of the cost co-efficient in the objective function of a LPP associated with both basic and non-basic variables.
33. Find the optimal solution of the LPP:

Maximize $\quad z=4 x_{1}+5 x_{2}$
$3 x_{1}+4 x_{2} \leq 14$,
$4 x_{1}+2 x_{2} \leq 8$,
$2 x_{1}+x_{2} \leq 6$,
$x_{1}, x_{2} \geq 0$.

Show that the optimality of the solution is not violated if the right hand side of the first constraint varies between 6 and 16 . Show further that the range of $c_{2}$ is $\left(\frac{5}{2}, \frac{20}{3}\right)$ in order that the optimal solution obtained remains optimal.
34.

What is a pseudorandom
number?Write the application areas of it. Use the middle-square method to generate five random numbers using $x_{0}=3043$.
35.

Use the linear congruence method to generate 20 random numbers using $a=5, b=3$, and $c=16$. Comment about the results of each sequence. Was there cycling? If so, when did it occur?
36.

Using Monte Carlo simulation, write an algorithm to find the area trapped between the two curves $y=x^{2}$ and $y=6-x$ and the $x$ - and $y$-axes.
37. Discuss the sensitivity of variations in the requirement vector of a standard LPP
a) Find the optimal solution of the LPP

Maximize $\quad z=4 x_{1}+3 x_{2}$

$$
x_{1}+x_{2} \leq 5
$$

Subject to

$$
3 x_{1}+x_{2} \leq 7
$$

$$
\begin{gathered}
x_{1}+2 x_{2} \leq 10, \\
x_{1}, x_{2} \geq 0 .
\end{gathered}
$$

Show how to find the optimal solution to the problem if
i)

The first component of the original requirement vector be increased by one unit, and the third component be decreased by one unit;
ii)

Two units decrease from the second component of the original requirement vector.
iii)
38. a. Use the convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+1)\left(s^{2}+1\right)}\right\}$.
b. Apply the convolution theorem to prove that

$$
B(\mathrm{~m}, \mathrm{n})=\int_{0}^{1} u^{m-1}(1-u)^{n-1} d u=\frac{\mathbb{T}(m) \mathbb{T}(n)}{\mathbb{r}(m+n)}, \mathrm{m}>0, \mathrm{n}>0 .
$$

39. Solve $\left(\mathrm{t} D^{2}+(1-2 \mathrm{t}) \mathrm{D}-2\right) \mathrm{y}=0, \mathrm{y}(0)=1, y^{1}(0)=2$, where $\mathrm{D} \equiv \frac{d}{d x}$.
40. Evaluate $\mathrm{L}\left\{\int_{0}^{t} \frac{\sin u}{u} d u\right\}$ by the help of initial value theorem.
41. Solve the following LPPs using the simplex method:

$$
\begin{aligned}
& \text { Maximize } z=10 x_{1}+x_{2}+2 x_{3} \\
& x_{1}+x_{2}-2 x_{3} \leq 10, \\
& \text { Subject to } \\
& 4 x_{1}+x_{2}+x_{3} \leq 20 \text {, } \\
& x_{1}, x_{2}, x_{3} \geq 0 \text {. }
\end{aligned}
$$

42. Make a graphical representation of the set of constraints of the following LPP. Find the corner points of the feasible region. Then solve the problem graphically.

$$
\begin{aligned}
& \text { Minimize } z=4 x_{1}+2 x_{2} \\
& \\
& \qquad \begin{array}{c}
3 x_{1}+x_{2} \geq 27, \\
\text { Subject to } \quad-x_{1}-x_{2} \leq-21, \\
x_{1}+2 x_{2} \geq 30, \\
\quad x_{1}, x_{2} \geq 0 .
\end{array}
\end{aligned}
$$

43. Solve the Legendre differential equation of the form

$$
\left(1-z^{2}\right) \frac{d^{2} y}{d z^{2}}-2 z \frac{d y}{d z}+n(n+1) y=0 .
$$

44. Examine that whether infinity is a regular singular point for Bessel's differential equation or not.
45. Using Laplace transformation, find the solution of initial value problem

$$
\frac{d y}{d t}+y=0, \frac{d y}{d t}-x=0, x(0)=1, y(0)=0
$$

46. Find the Laplace transformation of $t \frac{d^{2} y}{d t^{3}}+\frac{d y}{d t}+t y=0, y(0)=1, y^{\prime}(0)=0$
47. (a ) Using the Laplace transformation, find the solution of
$\frac{d^{2} y}{d x^{2}}-4 \frac{d y}{d t}+3 y=1, y(0)=y^{\prime}(0)=0$
(b) Evaluate by Convolution theorem $L^{-1}\left[\frac{1}{(s-2)\left(s^{2}+1\right)}\right]$
(c) Examine that whether infinity is a regular singular point for Legendre differential equation or not.
48.(a) Solve the Legendre differential equation of the form
$\left(1-z^{2}\right) \frac{d^{2} y}{d z^{2}}-2 z \frac{d y}{d z}+n(n+1) y=0$.
(b) Solve the Bessel differential equation of the form

$$
z^{2} \frac{d^{2} y}{d z^{2}}+z \frac{d y}{d z}+\left(z^{2}-n^{2}\right) y=0
$$

49.(a) Solve the Legendre differential equation of the form

$$
\left(1-z^{2}\right) \frac{d^{2} y}{d z^{2}}-2 z \frac{d y}{d z}+12 y=0
$$

(b) Solve the Bessel differential equation of the form

$$
z^{2} \frac{d^{2} y}{d z^{2}}+z \frac{d y}{d z}+\left(z^{2}-36\right) y=0
$$

50. Show that when n is a positive integer, $J_{n}(x)$ is the coefficient of $z^{n}$ in the expansion of

$$
\operatorname{xp}\left(\frac{x\left(z-\frac{1}{z}\right)}{2}\right)
$$

51. Prove that for the Bessel's function $2 J_{n}(x)=J_{n-1}(x)-J_{n+1}(x)$
52. Establish the Bessel integral equation.
53. Solve the following LPPs using the simplex method:

$$
\begin{gathered}
\text { Maximize } z=10 x_{1}+x_{2}+2 x_{3} \\
\qquad \begin{array}{c}
x_{1}+x_{2}-2 x_{3} \leq 10 \\
4 x_{1}+x_{2}+x_{3} \leq 20, \\
x_{1}, x_{2}, x_{3} \geq 0
\end{array} \\
\text { Subject to }
\end{gathered}
$$

54. Show that when n is a positive integer, $J_{n}(x)$ is the coefficient of $z^{n}$ in the expansion of $\exp \left(\frac{x\left(z-\frac{1}{z}\right)}{2}\right)$.
55. Prove that for the Bessel's function $2 J_{n}^{\prime}(x)=J_{n-1}(x)-J_{n+1}(x)$
56. If $\alpha$ and $\beta$ are the roots of the equation $\mathrm{J}_{\mathrm{n}}(\mathrm{z})=0$, then show that $\int_{0}^{1} z J_{n}(\alpha z) J_{n}(\beta z) d z=\left\{\begin{array}{lr}0, & \alpha \neq \beta \\ \frac{1}{2}\left[J^{\prime}{ }_{n}(z)\right]^{2}, \alpha=\beta\end{array}\right\}$
57. Prove that $\quad P_{n}(x)=\frac{1}{2^{n} n!} \frac{d^{n}}{d x^{n}}\left\{\left(x^{2}-1\right)^{n}\right\}$

END

